INSERVICE TEACHER EDUCATION IN MATHEMATICS: EXAMINING THE INTERACTION OF CONTEXT AND CONTENT[1]

Deborah Loewenberg Ball and Sandra K. Wilcox[2]

School mathematics is widely criticized for its emphasis on memorization and algorithms (e.g., Good, Grouws, and Ebmeier 1983; Goodlad, 1984; Madsen-Nason and Lanier, 1987; Stodolsky, 1988; Wheeler, 1980); considerable evidence suggests that few students develop conceptual understanding of mathematics in school (e.g., Dossey, Mullis, Lindquist, and Chambers, 1987; National Research Council, 1989). Consequently, the practice of mathematics teaching is currently the target of various improvement efforts. Although a number of these efforts focus on the teacher and make staff development the vehicle, little is known about the effects of different approaches to inservice education in mathematics. In particular, we lack insight into how the *context* of the staff development interacts with its *content* to affect what teachers learn and subsequently do in their classrooms.

Investigating the effects of inservice teacher education demands that researchers take a close look not only at the apparent effects on teachers' classroom practices but also at the inservice programs themselves. This paper illustrates an approach to analyzing the content and contexts of inservice teacher education programs. We compare inservice programs in mathematics for elementary teachers. One program is part of a large urban school district's initiative to revise its mathematics curriculum; teaching this curriculum is mandatory for the district's teachers. The other program is conducted by a local college; participation is thus voluntary and selective and participants come from a number of demographically varied school districts. Despite these differences, the two programs appear to share the goal of teaching mathematics for conceptual understanding. They also face some predictably similar obstacles in attaining that goal: traditional views of mathematics and mathematics teaching and learning, the organization of schools and the conditions of elementary school teaching, and the knowledge and skill of elementary teachers.

The analysis draws on interviews with staff and teachers in each program, program documents, as well as on observations of workshops and sessions. We describe aspects of each program's *context*: factors of organization, surroundings, time and timing that contributed to shaping the conditions for

[1]An earlier draft of this paper was presented at the annual meeting of the American Educational Research Association in San Francisco, March 1989.

[2]Deborah Ball, an assistant professor of teacher education at Michigan State University, is a senior researcher with the National Center for Research on Teacher Education. Sandra Wilcox, an MSU instructor in teacher education, is a research assistant with the NCRTE.

teachers' learning. We examine two sets of contextual factors: the settings of the inservice sessions themselves and the settings of the participating teachers' practice. Despite very obvious differences in the contexts of each of the programs, they shared a common goal: to increase participating teachers' capacities to teach mathematics for conceptual understanding. In examining program *content*, which we define as the enacted curriculum of the program, we focus on the programs' goal of teaching mathematics for understanding--what each program meant by this goal, what they thought teachers needed to know in order to teach for understanding, as well as their assumptions about how teachers would learn this. The paper concludes with a discussion of the interaction of context and content in inservice teacher education and raises questions critical for researchers interested in the effects of different approaches to working with practicing teachers as well as for policymakers and researchers intent on changing practice.

Inservice in an Urban District

Program Background

The first inservice program is part of a large urban school district's initiative to revise its mathematics curriculum. In January 1987, the superintendent and the board adopted a K-5 mathematics curriculum and mandated its citywide implementation over a three-year period beginning with the 1987-88 school year. The analysis in this paper draws on data from a larger study that investigated how and to what extent political, economic, and organizational factors shape the reform of curriculum and mandates for its implementation, using mathematics as the case (see Wilcox, 1989). The study was conducted during the first year of the implementation effort. In this study, the researcher observed nearly 50 hours of teacher inservice and conducted numerous interviews with school personnel and community people. The documents collected included curriculum materials; district records of student achievement, drop-out rates, and student mobility within the district; reports of the federal desegregation monitoring commission; published reports by public and private agencies on the city's schools; and demographic and economic data for the city.

Described as "conceptually based," the curriculum was organized into 10 mathematical strands--calculators and computers, estimation and approximation, functions and relations, geometry, measurement, numeration, operations, patterns, probability and statistics, and sets and logic--that were spiraled within and across the grades. At each grade level, instruction was sequenced by a series of 100 or more objectives organized in sets under one of the 10 strands. The inservice program aimed to acquaint teachers with and help them implement the newly mandated curriculum strands and objectives. The district's two elementary mathematics supervisors planned the inservices. Our analysis draws on data from the inservice conducted by one of the supervisors, Marilyn Miller,[3] during the 1987-88 school year.

Context: The Setting of the Program

The inservice consisted of a series of three after-school workshops. Each series of workshops was held in a single week on Monday, Tuesday and Thursday from 4:00-7:00 p.m. The inservice was organized by grade level--K-1, 2-3, 4-5--and by geographical area. Miller conducted a total of nine weekly workshops. She conducted six at an educational center at a local private college. To attend many teachers had a considerable drive from their schools. She held the other three in a centrally located middle school.

Teachers self-selected to participate and were paid the contractual stipend of $12.64 per hour. Attendance at each workshop series was limited to roughly 45 participants. While the total number of participants preregistered for Miller's nine workshops was near 400, the number who attended was less than 300.[4] Several principals and assistant principals also took advantage of these inservice opportunities.

At the conclusion of the first series for K-1 teachers, Miller expressed extreme dissatisfaction with the workshop design.

> This scheduling is bad. These teachers are already tired when they come after a full day at school. And this way doesn't give them time to think about what we have done or try some things and then come back and talk about it. You need more time to pound in this conceptual development. I'm not going to agree to this next time. I'm tired too by the third day.

She was not alone in this assessment. Some teachers found it difficult to muster the energy to thoughtfully engage in some of the activities. One exercise called for teachers to read two short articles on research in mathematics and then tell a partner what they had learned. At one workshop, a teacher rolled her eyes, stifled a yawn, and commented to a colleague at her table, "These articles are just too heavy for this time of day. I don't want to do this. I'm just going through the motions." Others at her table agreed.

Some teachers who chose not to attend the workshops said they made their decision based on when the inservice was offered. As one teacher put it,

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[3]This is a pseudonym.

[4]In October 1987, the district employed 3612 teachers to staff 163 elementary schools. Every staff member's request to participate in this inservice was honored. The discrepancy between preregistration and attendance was partly explained by the fact that four of the workshops were held in December in the two weeks prior to the holiday break. Teachers who did attend reported that this was an especially "hectic" time in their buildings, given the additional activities for observing the holidays.

> Why are teachers expected to attend these meetings on their own time? If this is so important why don't they provide for this during the school day? They send teachers to EEEI training and give them five full days of released time.[5] This just says to me that they don't think this is all that important.

There was an additional complication for those who might otherwise have been interested in the inservice. Some schools were on a late schedule to accommodate student bussing. As a consequence, teachers were not released from their buildings until after 3:45, making attendance at a 4:00 workshop held elsewhere difficult, at best.

Context: The Settings of Participating Teachers' Practice

This inservice effort needs to be understood within a broader set of contextual factors that had an impact on the working conditions of teachers in the district. Nearly two decades of industrial dislocation and economic disinvestment from the city had significantly reduced the district's ability to provide a quality education program. Teachers universally complained about the lack of materials, especially textbooks and paper. The new curriculum embraced the use of calculators in the mathematics classroom, yet not a single teacher who attended the workshop series had a set of calculators for her/his students.

The social dislocation that resulted from structural changes in the city's economy was also felt in the classroom. The dramatic increase in the ranks of the city's chronically unemployed, the growing number of homeless families with children, and the forced moves of families within the city to secure temporary housing meant that many children attended several different schools in one year. For many teachers this meant that a significant subset of students in their classes changed over the course of the year. In some cases, only half the classroom membership remained stable. The decision to adopt a citywide mathematics curriculum was, in part, a response to the problem of multiple transfers within the district.

Structural changes in the economy of the city over which schools had no control were coupled with significant changes in the district's organizational structure. Following 12 years of decentralized, semiautonomous regional administration, the district was recentralized in 1981. The reconfigured district was organized into areas, each with an administration with significantly reduced authority. Teachers found themselves in an unenviable position. There remained considerable ambiguity about who was in charge and whose directive they were bound to. While the central administration was telling

[5] EEEI--the Essential Elements of Effective Instruction--is the district's version of Madeline Hunter's generic teaching behaviors associated with "effective teaching." The district has made a commitment to train all instructional and supervisory staff in EEEI, known locally as "triple E I."

teachers to follow the sequence of instructional objectives that accompanied the new curriculum "in the order listed," some area and building administrators were ordering teachers to concentrate on a narrow set of computational skills assessed by the various tests administered citywide.

Teachers had been mandated to implement citywide curricula in *seven* content areas--language arts, social studies, mathematics, science, music, art, and health/physical education. This initiative represented not only an attempt to equalize learning opportunities across the district but was a response to considerable pressure to increase student scores on state-administered tests of educational achievement. The mandate also reflected a continuing effort by central administrators to further reduce the authority of area superintendents, exert increasing control and centralization over curriculum, and fashion curriculum in a way that supported instruction to improve test scores. One central administrator did acknowledge that there were "risks" in this curriculum reform agenda. As he put it, "There is the risk that implementation will just be mechanical and that we are taking away decisions of teachers. But when things aren't working, when the district is under pressure, there is the tendency to tighten up."

Teacher response to the mandate was mixed. Some welcomed the reshaping of mathematics content around strands as evidence that the district was moving away from a "back-to-basics, minimum competency, teach-to-the-test" orientation. Others felt the content had not really changed, that the strands were "just fancy names for things we have always taught." Others felt some of the new objectives were "too sophisticated for the kinds of kids we get." Regardless of their opinion about the appropriateness of the new mathematics curriculum, the majority who attended inservice felt overwhelmed by the demand to implement simultaneously newly mandated curricula in *all* content areas. And some displayed skepticism about the district's long-term commitment to the new curriculum. At one inservice, a teacher questioned Miller:

> How long are we going to do this [use the strands and objectives]? Are we going to be doing something else in three years? I'm trying to learn this but if I do are you just going to snatch this away in a couple of years and then we'll have to learn something new?

Many teachers wanted assurances that their efforts to become comfortable with the new curriculum strands and objectives and competent in following the instructional sequence would not be met by a new and different initiative a few years hence.

The Content of the Program

Teaching mathematics for understanding. Miller had three main objectives for the citywide inservice:

> First, I want to develop their understanding of what conceptual understanding is--the use of manipulatives and hands-on activities with lessons. I want them to become familiar with the research on effective math instruction. And I want to introduce them to cooperative learning, give them an opportunity to work in groups to see how that arrangement contributes to concept understanding.

The meaning that was attached to teaching for "conceptual understanding" was evidenced by the ways in which Miller talked about mathematical concepts and concept development, what she believed teachers needed to do to teach for understanding, and how she modeled teaching for conceptual understanding in workshops.

Miller, to use her own words, was "very model-oriented." At every point where she talked about "teaching a concept," the notion of modeling was key:

> Teaching for conceptual understanding is the most important process. It is not unique to math but it is the grindstone of learning. . . . I am very model oriented--this is the model I want to see. The second day of the workshop we talk about concept understanding and I model it.

Model here had three meanings. First, Miller believed that teachers should introduce a mathematical idea with a manipulative that is a concrete representation--a model--of the idea. As she put it,

> Everything [in math] is a concept and you need that before other processes. Like you need to understand numeration before addition and before the algorithm. For every topic there is a model to develop that concept. Take place value and regrouping. Base-10 materials are better than bundles of sticks because you can't take 1 from the 10 strip. It is hard to find the model sometimes but I know there is one. Then you need to go to the pictorial so that when kids are led to the symbolic they have a picture in their memory bank.

A second meaning Miller attached to the term model referred to the process by which she believed students learned concepts. Miller introduced teachers to the Rathmell Model for Mathematics Concept Development. The model specified five steps that required multiple opportunities for children to (a) generate the idea and give multiple examples; (b) recognize instances of the idea--what it is, what it is not; (c) represent the idea in three forms--a concrete model, oral language and written symbols; (d) represent the idea from one form to another; and (e) learn the properties of the specific concepts. Her example to elaborate the process was how to teach children the concept of the number 5.

Model also referred to lesson design and a mode of teaching the lesson. Miller described planning a lesson in the following way:

> I want to have teachers appreciate the need for more lesson development when they are planning to teach a concept. Look at the concept. Know what they are getting across. What is the model and what modeling needs to be done and what are the steps I need to take to model a concept. What are the questions I should ask to get at understanding. What materials do they need to practice with. . . . Teachers should have a single objective in mind. It should be clear and stated to the learner in a way so they know what they are expected to learn. The focus of the lesson should be on that one specific piece of information.

Once a lesson had been planned, Miller believed there was a particular sequence that instruction should follow. She called it "the model for effective math instruction." The instructional components were (a) mental arithmetic--five minutes of mental computation at the beginning of class; (b) motivation--getting students into a "mathematical mode," recalling previous learning and relating it to the day's lesson; (c) lesson development or instructional input--a minimum of 20-25 minutes where the "*teacher is teaching*," demonstrating with manipulatives that model the concept or process, asking product and process questions to check for student understanding; (d) guided practice; (e) independent practice; and (f) closure--students say what they have learned from the day's lesson. Miller told workshop participants, "This is a model of instruction based on research that has identified teacher behaviors that lead to increased student achievement on standardized tests."

Focal strategies in teaching for understanding. The first day of the three-day inservice was devoted to two main topics; defining the mathematical strands that organized the newly mandated curriculum and introducing a specific model of cooperative learning. Miller told teachers, "Over 700 research studies support this structure for learning. . . . Cooperative learning leads to higher achievement, more intrinsic motivation, better attitudes toward teachers and the school, higher self-esteem, peer group support, more on-task behavior and better collaborative skills."

The content of the second day was a curious mix of several topics: (a) a "cooperative logic activity" that led to a discussion about cooperative learning and problem solving strategies; (b) "jigsawing," where each teacher read two articles from *Research Within Reach* on estimation and mental arithmetic, problem solving, and manipulatives and then "taught" others at the table what she/he learned; (c) the introduction of "five learning processes--technological applications, algorithmic procedures, mental strategies and estimation, problem solving, and concept development;" (d) and introduction to the Rathmell Model of Concept Development.

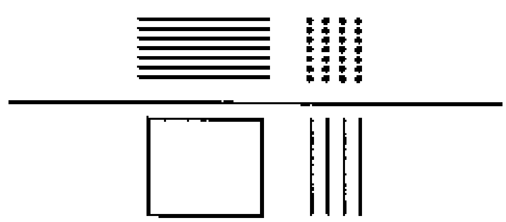
Day three was devoted almost exclusively to "effective mathematics instruction." Miller made reference to the work of Edmonds, Hunter, and Good and Grouws. She gave teachers a test to assess their knowledge of teacher behaviors that were claimed to result in increased student achievement. As her final activity, Miller introduced the components of an effective mathematics lesson and demonstrated by teaching a model lesson.

The model lesson for K-1 teachers was devoted to "trading ten ones for one ten" using base ten materials. The entire lesson was conducted with concrete materials only. As Miller told the participants, "This kind of work is necessary for the addition algorithm. You can also see that there is no reason to teach regrouping separate from non-regrouping."

For upper elementary teachers, Miller worked with base-10 materials in a somewhat different context. Rather than teach a model lesson devoted to a single objective, she demonstrated how these materials could be used to model and teach the four conventional algorithms of arithmetic. As she manipulated the materials for addition and subtraction, making trades, she wrote the problem symbolically saying,

> Writing this with numbers is simply making a record of what we have done with the concrete materials. This is what the algorithm means. You're showing them what this process means. Many times kids don't know what this means. They've heard the rules. Now they know why the regrouping actually took place. . . . Another thing you could do is make a record with pictures. A large square could represent a 100, a bar could represent a 10, and a small square could represent a 1. So if they don't have the concrete materials, they can draw representations of the problem and show the trading.

Miller manipulated the arrangement of the base-10 materials to replicate exactly the procedural steps in the conventional long multiplication and long division algorithms. To model "17 x 14," teachers were instructed to start with "10 x 10 " (which they represented with a hundreds square) and then "build up and out," creating the array shown below. Then they were instructed to draw a horizontal line above the hundred square:



As Miller wrote a record of what they had done, she told them,

> Above the line we have seven rows with 14 in each row. We have seven 10s and we have seven rows of 4. We can trade twenty of the 1s for two 10s and have eight 1s left. So seven 4s is 28, we trade for two 10s, write the 8 and carry the 2 which is combined with the seven other 10s giving us nine 10s. Below the line we have ten rows with 14 in

each row. Ten times fourteen is 140.

At the overhead she wrote,

$$\begin{array}{r} 2 \\ 14 \\ \underline{17} \\ 98 \\ \underline{140} \\ 238 \end{array}$$

Teachers seemed very confused about how to arrange the concrete materials, which number represented the number of rows and which told the number of elements in each row. Noticing their confusion but aware that little time remained, she said,

> You're going to have to practice this before going to kids. Be sure and plan the exact problems you are going to use with them. You can see I have my cards. I always know exactly what problems I'm going to do whether it's with teachers or kids.

And one teacher responded loudly, "I guess so!!!"

In three days of inservice, teachers learned about the

* 10 strands of the K-5 mathematics curriculum.
* 3 goal structures for learning in the classroom.
* 4 elements of cooperative learning.
* 3 rules for "groups of four" in cooperative learning.
* 6 outcomes of cooperative learning.
* 5 mathematical learning processes.
* 5 steps to concept understanding.
* 4 characteristics of a problem situation.
* 4 steps in problem solving.
* 16 problem-solving strategies.
* 11 behaviors that characterize effective teachers.
* 11 components to planning and delivering an effective mathematics lesson.

The SummerMath for Teachers Program

Program Background

The second program examined in this paper is SummerMath for Teachers' Educational Leaders in Mathematics Project (ELM), a local college- and NSF-funded inservice program for elementary teachers in mathematics. Although the program still exists, program staff are continually revising and adjusting its content and structure; this analysis draws on data gathered from 1987-1988 as part of a larger study of teacher education. In this study, researchers followed participating teachers over time to learn their reactions to the program and to explore its effects on their practice. Researchers interviewed and observed these teachers at intervals over the course of more than two years, beginning the spring before they began the program, throughout their participation in ELM, and after they had completed the program. Researchers also documented program sessions and other work with teachers and interviewed program staff to learn about their purposes and rationale for what they were doing (see National Center for Research on Teacher Education, 1988, for more information about this study).

Interested teachers applied and were admitted to the program from several school districts within a 40-mile radius of the college. A total of 24 teachers per year were accepted into the program.[6] In general they attended the program in pairs from a school district because the program director believed that "teams have more impact."

According to program staff, the participating teachers' mathematics background was, in general, "deplorable." Mathematics was, to them, computation and facts, and they tended to be anxious and to think they were not "good at math." Still, they were experienced (with over 16 years of teaching experience, on the average) and seemed to the program staff to be strong and dedicated teachers.

Teachers' participation was entirely voluntary. Reasons for attending varied: Some teachers came because they thought they would learn about using computers in the classroom, others wanted to increase their own confidence with mathematics, others wanted to organize their math program differently. In general, they did not seem to be seeking radical change in their practice, nor did there seem to be a push from their local school districts to do so. District administrators were asked to commit in writing that participating teachers would be allowed to purchase manipulative materials (e.g., base-10 blocks, Unifix cubes), that they would have regular and easy access to an Apple computer for their class, and that some of their extra duties would be reduced in order to allow them additional time for planning. In fact, the commitment to purchase manipulatives was the only one of the three that was consistently adhered to.

Context: The Setting of the Program

ELM had two components, beginning with a two-week residential summer institute, succeeded by a year of intensive individual classroom follow-up to teachers in their own classrooms. During this summer institute, which teachers described as "intense," staff led activities designed to help teachers to take a new look at the learning and understanding of mathematics and, consequently, at the teacher's

[6] An additional 12 teachers were admitted each year from around the United States. These teachers participated in the summer institute only (see below).

role in fostering learning.

The summer institute took place in midsummer in a beautiful old dormitory on the college's campus where teachers slept, ate, and worked. The atmosphere was one of a teachers' summer camp, filled with camaraderie and spirit, but intense. Group activities were held in a massive hall consisting of three huge rooms. Dining tables were set up in two of the rooms and the class tables (round wooden tables) were set up in the third. Chandeliers hung from the high ceilings. Meals were served buffet-style; the food was widely acclaimed as bountiful and good. The dorm also had several lounges (living rooms) with lovely upholstered sofas and chairs in which teachers sat and talked, or found a quiet spot to write in their journals. Outside the front door, the smokers congregated at breaks and free time, talking and puffing. In one of the lounges, the program had set up a library of teaching resource materials where teachers were often seen browsing.

Each day began with breakfast at 7:00 a.m. Mornings were focused on mathematical problem solving; the afternoon schedule included a hour and a half of physical education (dance and tennis) followed by a two-hour Logo class. Following dinner, teachers wrote in their journals, pursued unsolved problems from the morning's activities, or worked on computer programs. Occasionally guest speakers made evening presentations (e.g., on issues related to women and mathematics).

Both teachers and staff agreed that the summer institute was, for many participants, an intense and, often, emotional experience. Tears were not uncommon. One of the program directors explained that "there is a strong psychological component" involved in asking experienced and successful teachers to

> take a hard look at what they do in their classrooms and be willing to see that perhaps what they have been doing for 42 years, or 30 years, or 10 years, is not as effective as it might have been. There are major changes that could be made to make it much more effective. That's a tremendously anxiety-and dissonance-producing situation.

Interviews with teachers provided a glimpse of this. One teacher summarized the other participants' reactions: "People were really getting very upset--very uptight." Another teacher gave a personal story. Before the summer institute, she said she had "felt on top of the world" about herself as a teacher. She thought she was a good mathematics teacher, that she did things that helped her students be successful with math. After several days at the summer institute, she felt "down below the dirt." She said she was worried about "all those 18 years of teaching, all those kids." With tears in her eyes, she now worried that her students had never really understood the math, the concepts she was trying to teach. She said she was glad that, at least, she had used some manipulatives. Several teachers described the experience as intensely recharging and personally as well as professionally stimulating. One teacher said it was just what one had been looking for, and another commented that she could not wait to go back to teaching math in the fall to build on what one had learned in the summer.

Starting about three weeks after school opened in the fall, ELM staff members visited the teachers every week in their classrooms. Their role was to support the teachers' efforts to implement what they had learned in the summer institute. The staff person either taught a demonstration lesson or observed the teacher's lesson. Then the teacher and the staff person conferred for half an hour, discussing the lesson and other issues that might be of concern to the teacher. The teachers decided what they wanted to work on. Program staff urged them to "find ways that [they] can do a small enough piece that feels comfortable for them to start." One of the staff explained,

> If they set their goals too high, they'll get too discouraged and probably do nothing. We try to engender a healthy respect for how difficult this all is and how we are at the fairly early stages of learning how to do this.

These weekly consultations did seem to urge teachers to act in the ways the program stresses--to ask probing questions, to use manipulatives, to emphasize problem solving, for instance. One teacher commented that having the project director come out to her classroom every week provided motivation not to fall back into doing "the same old thing." The staff member's role was to ask questions to encourage self-evaluation, as well as to offer his/her own comments on the lesson. Program staff had no connection with building principals or other district administrators; their function was independent of the school district evaluation process.

In addition to the individual classroom consultations, ELM teachers came back to the college campus four times during the year for a day of discussion and work with the other participating teachers and the program staff. They exchanged stories about their work--its successes and rewards as well as its frustrations and disappointments.

Context: The Settings of Participating Teachers' Practice

The 24 teachers came from over 10 school districts in the local area. These school districts ranged from small rural consolidated schools to urban districts. Several teachers taught in small (e.g., six-room) schools with no building principal on site. Class sizes were small (e.g., 17) and the student population was homogeneously white and middle class. In contrast, several teachers taught in the diversely populated schools of a nearby city of about 160,000. Their classes were larger and included minority and limited-English speaking students.

Some districts had adopted mathematics textbooks that all teachers were expected to use; others used a criterion-referenced system for monitoring students' progress in mathematics. In some districts, teachers were free to choose the method and material of their teaching, although they felt bound to meet the expectations of teachers at the next level. They responded to implicit or explicit standards that their students know the multiplication facts, be able to set up a paper correctly with a heading, or know to label the answers to story problems. In none of the districts, however, was any initiative to alter the mathematics curriculum underway.

The Content of the Program

Teaching mathematics for understanding. ELM aimed "to help teachers develop their abilities to teach in a way that involves students in a problem-solving, active-learning approach to the learning of mathematical concepts." The focus was on developing teachers' notions about *learning*. Teachers, according to one of the program's directors, have no real theory of learning; they think about teaching and about what they do, not about what students do with it. Consequently, teachers are inclined to tell and show students how to do mathematics instead of creating activities that help students to construct understanding of the content. Teachers must "give up responsibility for getting the students to the answer."

ELM was based on a view of mathematics learning, labeled by the program staff as "constructivist," which holds that individuals must construct their own understandings of mathematical principles and concepts. As one of the program directors explained, "What `constructivism' means to us is that people don't take on meaning by hearing the meaning it has for someone else. They've got to have experience with the phenomenon to create meaning for themselves."

According to this view, students must be actively involved and their engagement must move from the concrete to the abstract levels if they are to develop conceptual understanding and the ability to solve mathematical problems. Applying mathematics to novel situations, inventing strategies, and assessing the reasonableness of one's solutions are among the hallmarks of understanding. Telling and explaining are less the teacher's trade in this approach. Instead, the teacher serves as a guide, facilitating students' learning by posing problems and asking questions aimed at helping students clarify their thinking (e.g., "What are you trying to do?" or "What does the ½ refer to here?").

Program activities involved teachers as learners in doing mathematics and solving problems, alone and in small groups. For example, over the course of two mornings in small groups, the teachers constructed numeration systems without relying on their familiarity with base-10 numeration. They wrestled with ways they might represent different quantities and how operations with those quantities might be symbolized and performed. On another day, they worked with volume, trying to figure out how many shoe boxes would fit in the hall where they were working. Staff members asked questions that probed the teachers' ideas as they worked (e.g., "What are you trying to do with that sketch?" or "What does this A represent?"); never did they show how to solve a problem or confirm an answer.

At times, staff members would ask teachers to step out of their role as learners to examine the pedagogy of the program. For example, on the day after the numeration activity was completed, one of the program's directors opened the discussion by saying,

> You've had the experience over the past three days of being math students and I daresay that what you experienced is at least somewhat different than what you experienced growing up. I would like for us as a group to pull together some ideas from that experience, what it has been like, some reflections on what it means to us, what sense we are making of it. I'd like to start out by having you pull out some of the characteristics you have noticed about the teaching. And when I say "the teaching," I'd like you to include the design of the lesson, the small-group activities, and the large group activities. What's *not* part of the math lesson is when we ask you to step *out* of the role of being a student and talk about the teaching and learning.

Teachers jumped into this discussion, commenting that they had been "guided, not led" to figure out solutions, that they had used manipulatives, that they had had opportunities to listen to others which helped them to expand their own ideas. One teacher commented that "the relationship between the teacher and the student is different from what we grew up with or what we are seeing in classrooms right now." Another teacher elaborated, saying that "in this setting, the teacher is fostering independence, questioning our *own* answers, how to go on on our own" instead of looking for "conformity."

Both project staff and participants remarked that staff members work with the participating teachers just as the program encouraged the teachers to work with their pupils. When asked about the view of teaching that underlies the program, the director said, "When I answer that question, I am also answering a question about the kinds of things we think about when we try to design our own lessons--and what we use to keep ourselves honest." According to teachers' written evaluations of the program, this feature of the program stood out to the teachers well. One teacher remarked that she had been to many other inservice programs where "people tell you lectures aren't any good--and then they sit there and lecture to you!"

An additional component of the program was a pair of physical education classes--a dance class and a tennis class. These took place after lunch every day and their purpose was to afford the participants another context in which to think about learning and teaching and about themselves as learners--of something other than mathematics.

Focal strategies in teaching for understanding. Working in groups, using nonroutine problems, using manipulatives (moving from concrete to abstract), asking probing questions were among the pedagogical strategies modeled. In addition, staff urged teachers to confirm right answers and use praise less--to fade out their role as authoritative source of knowledge.

Working in groups meant having students work on problems in pairs (or small groups) so that they can do more talking about mathematics. "Cooperative learning," however, was not mentioned as a label for this. One of the program's directors explained why working in groups for a significant portion of the class time was an important part of the program's view of teaching for understanding:

> My bias is that there should be a substantial amount of small group work, for two reasons: One is, if students are going to work individually--well, they can do that at home. . . . And, the second is (this is small-group versus whole group), there's just a lot less air time when one person's speaking versus 15. So I think it's an important tool for getting more participation.

Program staff also emphasized developing the ability to listen to students and to ask "probing"--instead of "leading"--questions. They acknowledged that most teachers become extraordinarily good at asking questions that lead students through the steps to the answers. When they do this, teachers imagine they are helping kids to do it on their own, when, in fact, all that students are doing is following the teacher's lead. ELM staff promoted the view that "the kid who figures something out for himself is better off and will remember it better in a year that the kid we gave the best explanation to." A leading question is one that implies a path or an answer; a probing question encourages students to reflect back on what they have already done or said.

According to program staff, beginning to ask more probing questions was a difficult shift for teachers to make. One teacher, early in the fall after the summer institute, talked about her efforts to make this switch. She said she was "trying to get them to feel the discord if things aren't working." She said that she sometimes asked questions that were very leading but that asking those kinds of questions was "*very* hard to avoid."

Connected to the importance placed on probing questions was the strategy of paraphrasing. Program staff frequently asked teachers to paraphrase what one another had said. Based on the view that people do not learn by listening to someone else's meaning, students were said to be able to clarify what they were hearing others say by paraphrasing others' explanations and comments, thus enabling them to construct meaningful understandings of what they hear. Asking students to paraphrase also would give teachers the opportunity to monitor what students were learning.

Closely related to probing questions, too, was the view that teachers should give up responsibility for praising students when their solutions are correct. Instead, teachers' comments to students should center on probing and helping them to expand or justify their thinking. One of the staff members explained to the teachers

> this notion of being an independent learner. As long as the idea is that the teacher is going to tell me or I'm going to look it up in a textbook, I don't think we're really creating independent learners. It may be more anxiety-producing for people who are

used to looking to the outside for approval.

And another staff member expanded:

> If a child asks you if this answer is right, and you say yes, you've *robbed* him of the real learning. It's a question of *when* you say good, not *if* you say it. Once you've probed for understanding, and you're sure that the child knows, *then* to say, "You've convinced me, that's terrific, what you said really made sense to me. Why don't you share it with the rest of the class?" But I'd wait until the last moment when I'm really sure that the child really knows it.

This view was summarized by the teachers as "no praise" and generated a lot of resistance and argument on their part. They felt strongly that learners require "pats on the back" and encouragement and were unconvinced that providing this kind of positive reinforcement could be problematic. One of the program's directors responded:

> I like to think of this big control panel with buttons we haven't even been turning before--more praise, less praise--we've just been turning. Now we find out there's another control in teaching we can adjust. I don't know anything in teaching that . . . you do [just] one way.

One final component of the ELM view of teaching for understanding was the principle of beginning with the concrete and moving to the abstract level. Concrete might include pictorial representations; it need not mean physically concrete. Still, manipulatives were central--especially base-10 blocks and Unifix cubes which most of the teachers were encouraged to buy. All learning was said to progress along a continuum from concrete to representational to abstract, from the "known to the unknown." This, according to one of the program's directors, was the essence of "constructivism." Starting with concrete materials or experience provides everyone with a common base from which to construct their understandings; when teachers do not do this, instruction is "biased in favor of those who have had the experience somewhere." Program staff tried to persuade teachers to abandon their assumptions that manipulatives are only for "slower learners" or younger children by having them work with concrete materials themselves. Discovering how helpful they were to their own learning, teachers saw that these materials were not, as one teacher commented, "baby stuff."

Inservice as Teacher Education: Comparing the Two Programs

We have examined two inservice programs, focusing our discussion on aspects of the context and content of each program. Having considered these two cases separately, we move to a comparison of the programs by analyzing them as opportunities for teacher learning. To do this, we use four frames that contribute to examining inservice as teacher education: For each program, we discuss (a) the view of teachers held by the program; (b) the program's assumptions about what teachers need to know, be able to do and care about: (c) the program's assumptions about how teachers learn; and (d) models of changing teachers' practice that underlie each program.

View of Teachers Held by the Staff of Each Program

The supervisor who provided the district inservice thought that teachers simply lacked essential research knowledge and the pedagogical skills derived from that research. Her efforts were aimed at equipping them with this essential knowledge. In contrast, the ELM staff did not see participating teachers as missing critical knowledge but, rather, as having orientations that were significantly different from those central to the program. The staff viewed the teachers as dedicated professionals whose focus over the years had been on teaching instead of on learning. Consequently, the teachers had no theory of learning because it was not something about which they had thought much. The teachers had habits, such as telling students how to do math, asking leading questions, and praising, that suited their current approach to teaching but that would not suit an approach based on a constructivist view of learning.

Assumptions About What Teachers Needed to Know,
Be Able to Do, Care About

There were interesting similarities in the two programs with respect to what teachers needed to know and be able to do. Both programs emphasized introducing new ideas at the concrete level, working in groups, and asking questions as effective pedagogical strategies. But the meaning attached to each of these strategies, the ways in which teachers were to use them, and the purposes they served were different. In the district-sponsored inservice, beginning with the concrete meant manipulating physical representations of the mathematical concept or procedure. Learning was portrayed as requiring a specific sequence of development: first manipulating concrete models, linking concrete models to pictorial representations, and then using pictorial representations as the bridge to the abstract. In ELM, although manipulatives were a central feature, beginning with the concrete could mean introducing an idea with a physical model or a pictorial representation. In ELM, working in groups might mean pairs of learners or small groups working together. The district inservice promoted a specific model of "cooperative learning" in which each learner in a group was responsible for a specific aspect of the group process.

One of the striking differences was in the role of questioning. ELM staff urged teachers to ask *probing* questions. Probing questions encouraged students to be reflective about what they had done, invited them to pursue further hunches and make convincing arguments, and facilitated a shift away from the teacher as the sole source of epistemological authority. In contrast, in the district inservice, teachers were told to ask *product* and *process* questions. This form of questioning asked students to tell what answer they got and how they got it. Process questions were asked at the end of work on a set of problems and served as simply another check of correctness. Authority for knowing continued to rest with the teacher who decided whether student explanations were sufficient.

Virtually untouched in the district-sponsored inservice was teacher subject matter knowledge. Although the supervisor acknowledged that many teachers, by their own admission, were uncomfortable with the content of some of the strands, attending to this teacher concern was not an objective. The ELM program did engage teachers in doing mathematics themselves; still, subject matter knowledge was not the focus of this program either. Given the constraints of time and resources, ELM staff chose to emphasize beliefs about and commitments toward learners and learning in general and on teaching skills that are associated with those ideas (e.g., asking probing questions to help pupils clarify their ideas).[7]

Assumptions About How Teachers Learn

The inservice providers in both programs used the program's content as pedagogy but in very different ways. Miller modeled in inservice what she expected teachers to do in their classrooms; tell students what they needed to know, demonstrate with concrete materials, give them opportunities to practice what they had been shown, ask product and process questions to assess understanding, and have students tell what they learned. The ELM staff, too, was proud of modeling what they wanted teachers to do with their students. They asked probing questions, engaged teachers in solving problems in small groups, and led discussions in which participants clarified meanings and key ideas. In the case of ELM, however, modeling the content did not seem to be all there was to it. In trying to change the teachers, to change the ways in which they thought, what they believed, and their dispositions to act in particular ways, program staff self-consciously tried to establish an environment in which teachers, as learners, were likely to revise their ideas and ways of thinking. In other words, the program's content informed its pedagogy.

The staff in each program believed that change took place over a long time period but set about it in very different ways. In ELM, the yearlong follow-up was included in recognition of this assumption. Although change was assumed to take longer than even that year, staff seemed to hope that, during that year, teachers might develop a sufficiently solid commitment and sense of what they

[7]Program staff began to notice that teachers whose own mathematics background was weak had difficulty selecting and posing worthwhile problems or asking questions appropriately. Consequently, SMT now includes a mathematics course for participating teachers.

were trying to do, that they could continue learning on their own from their own efforts to teach mathematics differently. In contrast, the district inservice included no explicit, planned follow-up. Miller told teachers she would be happy to respond to individual requests for classroom visits, school-site workshops, and resource materials. Inservice provided the techniques. What teachers needed was time in their classrooms to practice delivering instruction in the mode prescribed.

The two programs' attention to and assumptions about the context of inservice and its contribution to teacher learning differed substantially. ELM staff paid particular attention to providing an environment that would support participants, given the discomfort and anxiety involved in the fundamental changes the program was trying to promote. The beauty and comfort of the surroundings as well as the attention to food and recreation were intended to provide an atmosphere of community to support this process. In the district program, the context in which inservice was provided did not seem to be a significant consideration. The organization of three after-school sessions in a single week produced increasing levels of disengagement among many participants by the last session. Although Miller would have preferred to offer the sessions over a more extended period of time, she acquiesced to the recommendations of a senior colleague. The time for beginning a workshop overlapped dismissal time at several schools, thereby limiting attendance for those teachers who might otherwise have been interested. The distance teachers had to drive between individual schools and the inservice site reduced potential participation. In the district inservice, the context itself was a constraint on teachers' participation.

Underlying Models of Changing Teachers' Practices

The city-sponsored inservice program was intended to support the implementation of a curriculum reform in mathematics. This reform effort grew out of an interaction over nearly two decades of broader social, political, economic and organizational contexts. Three significant factors that contributed to the reform were the black community's drive to combat racism and gain control over the schools, the school system's massive economic and organizational problems, and the central administration's efforts to respond to community interests while also centralizing and standardizing curriculum.

The aim of the program was to give information and techniques to teachers that would promote much-needed reforms in mathematics teaching and learning in this context. Equipped with this new knowledge, teachers would return to their individual classrooms to use the ideas they had found persuasive. The inservice program thus embodied a quality of technical rationality. Teachers were presented with "systematic knowledge,"--specialized, scientific and standardized--that "ignore(d) complexity, uncertainty, instability, uniqueness and value-conflicts" (Schon, 1983; p. 39). Elements of control, prediction, and certainty exerted a powerful influence on the content of what teachers were given (see Giroux, 1981). That influence could be seen in the formulation of highly specific learning objectives that carefully systematized content (see Apple, 1979). It was evident in assumptions that teaching and learning proceeded best when content was divided into small pieces (see Bullough, Goldstein, and Holt, 1984). And it underpinned an approach where the ends of schooling--higher achievement as measured by scores on standardized tests--were assumed and the means to that end were a matter of instructional technique and technical competence.

This model, the research out of which it grew, and the practices it prescribed were presented as objective facts rather than as something to be questioned, analyzed, or negotiated. What was portrayed was a consensus model of research. Teachers were not told that there was considerable disagreement about the suitability of the direct instruction model to teach for conceptual understanding and higher order thinking (Peterson, 1988). The dichotomy between the expert (the researcher and the supervisor) and the practitioner (the classroom teacher) was maintained, limiting the possibility and impact of individual teacher autonomy or change.

The ELM program, like the district program, also aimed to produce change in the way mathematics is taught. ELM's approach, however, grew out of an entirely different agenda, one motivated by a concern for the algorithmic nature of most school mathematics teaching and the thin understandings developed by most students. Instead of giving teachers specific knowledge and skills, this program aimed to help teachers themselves change. Consequently, the program was marked with a kind of tension, one that pitted program staff members' strong belief in a constructivist view of learning against their equally strong concerns for changing teaching practice. The program staff recognized that teachers, like pupils, construct their own understandings and that, consequently, teachers would take away different things from the program.

Some might become engrossed with using manipulatives. Others might focus on working in groups. Some might incorporate these pieces of the program into their usual ways of thinking and teaching. Yet, the ELM staff was concerned with changing the way mathematics is taught. They wanted teachers to teach in ways that were substantially different from modal practice, involving students in a problem-solving approach to the learning of mathematical concepts and abandoning the role of telling and showing. Since the ELM staff believed that, in order for teaching to change, teachers themselves needed to change, not just use different tools, the constructivist approach was fraught with tensions and uncertainties. How would teachers change their practices in any significant way unless they themselves really reconsidered and changed their assumptions about learning and about their role? But how could the program staff get them to change? The changes they made would grow out of what they constructed from the experiences of the program.

Conclusions

In this paper we have not begun to address questions of what teachers learned from either of these efforts to change their mathematics teaching--what they came away thinking, being able to do, and caring about, as well as how any of that, over time, affected their classroom practice. We have, instead, argued that investigation of the relative impact of such inservice programs requires close examination of the programs themselves--of their contexts, their content, and of the interactions between the two that produce opportunities for teacher learning. That both of these inservice programs aim to change teachers' practices related to the teaching of mathematics, that both emphasize the importance of student understanding, and that both extol the virtues of small group work and concrete materials makes them look similar in intent. Yet in context, the curricula--the opportunities for teacher learning--lose their similarity. Contrasts in context and in pedagogy produced programs that were significantly different in opportunity and, one would predict, in outcome.

Yet, without examining the outcomes of each of these programs, we conclude this paper by highlighting a fundamental tension that, we argue, may be endemic to designing programs of inservice education aimed at changing teaching practices. Changing teaching, argues Cohen (1988), is difficult due to a host of factors related to views of knowledge and teaching embedded in this culture, the social organization of teaching practice, and the demands of change within the context of the nature of teaching. "Educational improvement," he writes, "becomes more difficult, on the average, as it becomes more attractive and adventurous. One way to reduce difficulty and risk in any of these practices is to simplify and clarify results" (p. 26). In the case of the two programs examined in this paper, difficulty and risk were reduced by selection of program focus.

Teaching involves weaving together various kinds of knowledge and skill with considerations of context. It is shaped by teachers' dispositions to act in particular ways under particular circumstances (Ball and McDiarmid, 1988). With its interest in change, each inservice program chose a specific aspect of teaching practice as its focus. The district-sponsored program focused on research-based teaching techniques, ELM on a theory of learning.

Choosing a focus for inservice makes sense; it suggests a strategy of engaging teachers in changing aspects of their practice by targeting a single aspect or ingredient of their teaching. A broader sweep, especially under conditions of limited time, seems likely to do little more than ripple gently the waters of practice. Yet choosing a singular focus has its drawbacks, for the interactions among the threads of teaching are not well understood. Focusing on techniques of teaching without, for instance, engaging teachers in considering their assumptions about learning, may prove a futile intervention. Take cooperative learning as an example. When teachers encounter this as a research-based pedagogical strategy, but do not reexamine their views about independent work, cheating, or the need for classrooms to be quiet and orderly, what "cooperative learning" will mean as part of their practice may be superficial and a far cry from its essence.

Similarly, helping teachers to reconsider their ideas about how learning takes place without engaging them in thinking in new ways about the subject matter they are teaching may have similar unforetold results. In mathematics, for instance, the tasks that teachers use or create are as much a function of their understanding of the subject matter as of their views of how children learn. Teachers who come to view learning as a process of individual construction may nevertheless create tasks that do not provide genuine opportunities to build paths into the subject matter, that elicit algorithmic thinking or opportunities to practice as opposed to occasions for figuring out something less routine or straightforward. Teachers who conceive of mathematics as linear and rule-bound are, after all, in a poor position to do anything else.

The tension in designing an inservice program between choosing a specific target of change on one hand and attending to the whole interwoven web that is teaching on the other seems endemic to the problems of changing teaching practices. How inservice programs choose to deal with this tension in the design of the program is a dimension critical to exploring and understanding the program's results.

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